**MATRICES**

**What is matrix**: is an operator

**Definition**: matrix is a rectangular array arranged in rows and columns.

* A square matrix is said to be diagonal matrix if non-diagonal elements are zeros.

if bij=0 when i!=j

* A diagonal matrix is said to be scalar matrix if diagonal elements are equal.

if bij=0 when i!=j ,

if bij=k when i=j

* A diagonal matrix is said to be identity matrix if diagonal elements are ones.

if bij=1 when i!=j,

bij=0 when i=j

* All the elements of matrix are zeroes, is called null matrix or zero matrix.
* 2 matrices are said to be equal if they are of same order(no. of rows and columns) and corresponding elements are same. then those 2 matrices are called identical( or equal) matrices.

**Transpose of matrix**: exchanging of rows and columns of a matrix. ( A=aij, A'=aji).

*Note*: Transpose of non-square matrix is not possible

**Symmetry:**

* if A=A', A is Symmetric matrix. (aij=aji)
* if A=-A', A is Skew Symmetric matrix. (aij=-aji)

**Matrices Operations**

* ***Addition***: A[aji]+B[bij]=C[aij+bij]
* ***Subtraction***: A[aij]-B[bij]=C[aij-bij]
* ***Multiplication***: Suppose A=(aij)mxn and B=(bij)pxq. if n=p,

then C=(AB)mxq. Cij=sigma for k=1 to n (aik)(bjk)

**Commutative**:

If 2x3=3x2, then it is said to be commutative.

But, multiplication in matrix is not commutative. i.e., AB! = BA

**Trace** of matrix is sum of diagonal elements.  possible for only square matrix.

Tr(A)=a11+a22+a33+ ... .. .. ann

tr(A)=tr(A')

tr(A+B)=tr(A)+tr(B)

tr(A-B)=tr(A)-tr(B)

tr(AB)=tr(BA)

tr(AB)!=tr(A)tr(B)

**Properties of Matrices:**

* if 2 rows and 2 columns are equal, then det=0
* if det A=0, then A is singular matrix.

**Different techniques of solving an equation:**

1. **Cramer’s Rule:**

**Rules: det!=0 & Matrix should be square.**

ax+by=r;

cx+dy=s;

det=(ad-bc)

det1= (replacing 1st column of A with constants(r,s))= (rd-sb)

det2= (replacing 2nd column of A with constants(r,s))= (as-cr)

x=det1/det

y=det2/det

1. **By the Inverse of matrix:**

A X = B

X = A(inverse)B

If A is 2x2 matrix, A(inverse)=(1/|A|)(A’cB)

**A (transpose cofactor) can be found by**

1. Traversing diagonal elements. i.e., a as d and d as a
2. Change the signs of non-diagonal elements. i.e., b as –b and c as –c

If A is 3x3 matrix, A(inverse)=[1/|A|]Adj(A)

**Adj(A) can be found by**

1. Find A’
2. Find dets of minor matrices of matrix and form a new matrix
3. Multiply the new matrix with alternate sign matrix.[(-1)^i+j]

**Consistent**:  if Rank(A)=Rank(A|B) ---> Unique solution

**Inconsistent**: if Rank(A)<Rank(A|B) ---> No solution

Rank says whether the equations have unique solutions, or infinitely many solutions or no solution or feasible or bounded.

**Rank**: max no. of linearly independent columns or rows of a matrix. Can be found using elementary row operations (or) if det(A)!=0, then Rank(A)=order of matrix.

**Gauss Jordan Method to find A(inverse):**

1. Form the augmented matrix by the identity matrix.
2. Perform the row reduction operation on this augmented matrix to generate a row reduced echelon form of the matrix.
3. The following row operations are performed on augmented matrix when required:

* Interchange any two row.
* Multiply each element of row by a non-zero integer.
* Replace a row by the sum of itself and a constant multiple of another row of the matrix.